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Now if we have regard to the order of  $\lambda, \mu, \nu, \xi$ , there are  $(p-1)^3$  ways of satisfying the original congruence. For each of three parameters can be assigned in  $p-1$  ways, and the congruence determines the remaining one. Hence we have

$$\begin{aligned}(p-1)^3 &= n_4 + {}_3C_4 n_3 + 6n_{2_1} + 2{}_2C_4 n_{2_2} + {}_4P_4 n_1, \\ (p-1)^3 &= n_4 + 4n_3 + 6n_{2_1} + 12n_{2_2} + 24n_1.\end{aligned}$$

Substituting the values found above for  $n_{2_2}, n_{2_1}, n_3, n_4$ , we get, when  $p \equiv 1 \pmod{4}$ ,

$$n_1 = {}_{2^{\frac{1}{4}}}^1 (p^3 - 9p^2 + 29p - 21)$$

and when  $p \equiv 3 \pmod{4}$ ,

$$n_1 = {}_{2^{\frac{1}{4}}}^1 (p^3 - 9p^2 + 29p - 33).$$

Hence, when  $p > 5$  is a prime of the form\*  $4l+1$  the number ( $N$ ) of sets of solutions of  $\lambda + \mu + \nu + \xi \equiv 0 \pmod{p-1}$  is,

$$\begin{aligned}N &= {}_{2^{\frac{1}{4}}}^1 (p^3 - 9p^2 + 29p - 21) + \frac{1}{2} (p-3)^2 + p-3 + p-5 + 4 \\ &= {}_{2^{\frac{1}{4}}}^1 (p^3 + 3p^2 + 5p - 9).\end{aligned}$$

But when  $p$  is of the form  $4l+3$ ,

$$\begin{aligned}N &= {}_{2^{\frac{1}{4}}}^1 (p^3 - 9p^2 + 29p - 33) + \frac{1}{2} (p-3)^2 + p-2 + p-3 + 2 \\ &= {}_{2^{\frac{1}{4}}}^1 (p^3 + 3p^2 + 5p + 3).\end{aligned}$$

## GEOMETRY.

284. Proposed by JOHN JAMES QUINN, Ph. D., Warren, Pa.

a) Suppose that two radii  $R$  and  $r$ , whose center is the origin, revolve with uniform angular velocities  $3\theta$  and  $\theta$ , respectively. What is the equation of the locus of  $P$ , the projection parallel to the  $X$  axis of the extremity of the radius  $r$  on the radius  $R$  produced if necessary.

b) Apply this curve to the trisection of an angle.

c) Suppose the ratio of their velocities is  $n\theta:\theta$ . Show how we can effect the multisection of an angle.

Solution by A. H. HOLMES, Brunswick, Maine.

a) Take  $O$ , the center of the circle, radius  $a$ , as the origin of coördinates. Then taking any angle  $\theta$ , we shall have  $r \sin 3\theta = a \sin \theta$ .

$\therefore r = \frac{a \sin \theta}{\sin 3\theta}$  is the equation of the locus of the point  $P$ .

b) Construct the curve  $r = \frac{a \sin \theta}{\sin 3\theta}$ . On the circumference of the circle

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\*The values  $p=3, 5$  constitute exceptions in the method employed. The final result does not hold for  $p=5$ . But, by inspection, when  $p=5$ ,  $N=10$ ; and when  $p=3$ ,  $N=3$ .

take an arc equal to any angle  $\psi$  from axis of  $x$ . Draw a line from the upper limit of the arc to the origin. From the point  $P$ , where this line cuts the curve  $r = \frac{a \sin \theta}{\sin 3\theta}$ , draw a line parallel to axis of  $x$  cutting the given arc. From this point draw a line to  $O$  making an angle  $= \frac{1}{3}\psi$ .

c) The equation of the locus of  $P$  would be  $r = \frac{a \sin \theta}{\sin n\theta}$ , and the multisection would be similar to the trisection.

NOTE. By projecting parallel to the  $y$ -axis Dr. Zerr obtains in a similar way the locus  $r = \frac{a \cos \theta}{\cos n\theta}$ , and effects the  $n$ -section of the angle in a manner similar to the above. Ed.

285. Proposed by G. E. BROCKWAY, Nashua, N. H.

Prove without the aid of the circle, that if the bisectors of the angles of a triangle be drawn, the greatest bisector falls on the least side.

I. Solution by ALFRED H. PARROTT, North Dakota Agricultural College, N. D.

Given scalene triangle  $ABC$ , and bisectors of angles  $A$  and  $C$ , supposing  $\angle C > \angle A$ . If  $\angle C > \angle A$ , then side  $AB > BC$ , and we are to prove bisector  $AD$  on side  $BC > \text{bisector } CE$  on side  $AB$ . If  $\angle C > \angle A$ ,  $\angle OCA > \angle OAC$  [If wholes are unequal, then halves, etc.].

$\therefore AO > OC$  [If angles of a triangle, etc.]. Now if  $OD > OE$ , the proposition is evident. But suppose  $OD < OE$ . Then on  $OE$  take  $OF = OD$ , and on  $OA$  take  $OG = OC$ , and draw  $FG$ . Then draw  $GH$  parallel to  $CE$ .

$\triangle FOG = \triangle COD$  [Three sides on one equal respectively, etc.]. Then  $\angle FGO = \angle DCO > \angle EAO$  [Halves of unequals, etc.].

Consider  $\triangle FGO$  and  $\triangle EAO$ ;  $\angle O \equiv \angle O$ ,  $\angle FGO > \angle EAO$  [Previous proof]. Therefore  $\angle GFO < \angle AEO$ .

A line drawn parallel to  $FG$  and through  $E$  will then intercept  $HG$  between  $H$  and  $G$  and  $HG$  is therefore  $> EF$ .

Now  $\angle AHG > \angle HAG$ , for  $\angle AHG = \angle AEC$ , [sides respectively parallel],  $\angle AEC = \angle B + \angle ECB$  [Exterior angle of a triangle equals sum of, etc.]. But  $\angle HAG < \angle ECB$ , and therefore  $< \angle B + \angle ECB$ , and hence  $< \angle AHG$ .

$\therefore AG > HG > EF$ , or  $AO - GO > OE - OF$ ,

$AO + OF > OE + GO$ ,  $AO + OD > EO + OC$ .

Hence  $AD > EC$ . Q. E. D.

